On Nonperturbative Calculations in Quantum Electrodynamics

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Abstract

A new approach to nonperturbative calculations in quantum electrodynamics is proposed. The approach is based on a regular iteration scheme for solution of Schwinger-Dyson equations for generating functional of Green functions. The approach allows one to take into account the gauge invariance conditions (Ward identities) and to perform the renormalization program.

The iteration scheme can be realized in two versions. The first one ("perturbative vacuum") corresponds to chain summation in the diagram language. In this version the exact result of two-dimensional Schwinger model is reproduced at the first step of calculations, but in four-dimensional theory the non-physical singularity (Landau pole) arises which leads to the triviality of the renormalized theory. The second version ("nonperturbative vacuum") corresponds to ladder summation and permits one to make nonperturbative calculations of physical quantities in spite of the triviality problem.

For chiral-symmetrical leading approximation two terms of the expansion of the first-step vertex function over photon momentum are calculated. The formula $f_2 = \alpha/(2\pi - \alpha)$ for anomalous magnetic moment is obtained (α is the fine structure constant).

For linearized equation of leading approximation a problem of dynamical chiral symmetry breaking is considered, the calculations are performed for renormalized theory in Minkowski space. In the strong coupling region $\alpha \geq \pi/3$ the results correspond to earlier investigations performed in Euclidean theory with cutoff: solutions arise with breakdown of chiral symmetry. For the renormalized theory a solution with breakdown of chiral symmetry is also possible in the weak coupling region $\alpha < \pi/3$, but with a subsidiary condition on the value of α which follows from the gauge invariance.

Introduction

The problem of nonperturbative calculations in quantum electrodynamics (QED) arose practically simultaneously with the principal solution of the problem of perturbative calculations which based on renormalized coupling constant perturbation theory. It is necessary to recognize, however, that the progress in the nonperturbative calculations during last decades is not too large. Quantitative description of nonperturbative effects either is based on non-relativistic foundations (an example is the bound state description based on non-relativistic Coulomb problem) or is rather open to injury for a criticism. Besides, the problem of inner inconsistency of QED exists (see [1]). This problem can be formulated as a deep-rooted thesis on triviality of QED in the nonperturbative region (see, for example [2], [3] and references therein).

The triviality means that the only non-contradictory value of the renormalized coupling is zero. Absence of asymptotic freedom in QED and unsuccessful looking for another type of self-consistent ultraviolet behavior are strong arguments in favor of the triviality[1]. An extremal expression of this point of view is a statement that QED can be treated exceptionally as the renormalized coupling constant perturbation theory, and, consequently, any nonperturbative calculations are excluded from the consideration. At a high account such a situation is not desperate since at very high energies the QED becomes a part of a grand unification theory which is based on non-Abelian gauge theory with the self-consistent asymptotically free ultraviolet behavior. However, it sounds rather unnatural that for a consistent relativistic calculation of positronium energy levels, for example, one should exploit a grand unification theory, while it is not need for calculations of annihilation cross-sections. It is difficult to find any physical reasoning for such a principal difference of these problems. Therefore a possibility to perform approximate nonperturbative calculations of physical quantities in the framework of QED itself without contradictions with the triviality of the exact theory (as it made in the perturbation theory) seems to be a necessary component of the theory.

At first sight the main problem of nonperturbative calculations in QED is an absence of a universal small parameter besides the fine structure constant. Due to this reason any partial summation of perturbative series seems to be an arbitrary procedure which can be apologized only by physical meaning of results. At the same time the absence of a small parameter itself is not an obstacle for using some approximation. A motivated approximation can give quite satisfactory results even without an explicit small parameter: examples are applications of variational methods or mean field type approximations in different areas of physical theory.

A general problem of various nonperturbative approximations in QED is consistent taking into account of requirements of the gauge invariance and renormalizability. It is clear that a necessary condition for such taking into account is the existence of a regular iteration scheme which in principle permits one to do an arbitrary large amount of steps toward an exact solution of the problem. In present work a such type scheme is proposed. The scheme is based on an approximation of Schwinger-Dyson equations (SDEs) for the generating functional of Green functions of QED by an exactly soluble equation. Its solution generates a linear iteration scheme each step of which is described by a closed system of integro-differential equations. The requirements of gauge invariance (in the form of Ward identities) are easily taken into account at each step of iterations. The renormalization of equations of each step is also not a principal problem. Note that equations for Green functions at leading approximation and at the first step of iterations look as familiar ones. Similar equations

[1] In QED with dynamical chiral symmetry breaking the situation can be different (see [4],[5]).

were written and investigated earlier in other contexts. A new thing is its appearance in the structure of the regular iteration scheme, and it is this circumstance that allows one to give them a successive quantum-field-theoretical interpretation.

Using a bilocal fermion source gives a possibility to formulate the iteration scheme in two versions. First of them on the language of Feynman diagrams of perturbation theory is analog of the summation of chain diagrams with fermion loop. This version is named "calculations over perturbative vacuum" since a unique connected Green function of the leading (vacuum) approximation is the free electron propagator. The calculations over perturbative vacuum for two-dimensional electrodynamics give the exact result of Schwinger model for a photon propagator as early as at the first step, but for physical four-dimensional case they lead to the appearance a non-physical Landau pole in the photon propagator and, as a consequence, to the triviality of the renormalized theory. Thus a practical meaning of the calculations over perturbative vacuum for four-dimensional QED becomes equal zero, though they grasp the main nonperturbative effect — the triviality of the full theory.

The second version of the iteration scheme can be compared on the diagram language with a ladder summation. This version is named "calculations over nonperturbative vacuum" since the electron propagator of the leading vacuum approximation is a solution of a non-trivial nonlinear equation. For this version of the iteration scheme the nonperturbative calculations become possible without contradiction to the triviality of full theory. The basic part of the work is devoted to the investigation of this version. Apart from the formulation of general states and principles of renormalization of Green functions, the article contains the calculation of the first-step vertex function (in the case of chiral-symmetric solution of the leading approximation equation). Also, for a linearized version of the theory the problem of dynamical chiral symmetry breaking is investigated.

A structure of the paper is the following: in the first Section the necessary notations and definitions are given; SDEs for the generating functional of Green functions are introduced in the formalism of bilocal fermion source, and generating relations for Ward identities are considered. In the second Section a general construction of the iteration scheme for solution of SDEs is given; also the leading approximation and the first step of iteration over perturbative vacuum are considered. In Section 3 the renormalization of the first-step equations is made; it is demonstrated that for the four-dimensional theory the calculations over perturbative vacuum lead to the trivial theory already at the first step. In Section 4 a scheme of calculations over nonperturbative vacuum is formulated. In Section 5 two first terms of expansion of the vertex function in photon momentum are calculated for chiral-symmetric vacuum. These calculations allow one to obtain a simple formula for anomalous magnetic moment: $f_2 = \alpha/(2\pi - \alpha)$, where α is the fine structure constant. Section 6 is devoted to investigation of dynamical chiral symmetry breaking in QED. In Conclusion a discussion of the results is given.

1 Schwinger-Dyson equations and Ward identities

We shall consider a theory of a spinor field $\psi(x)$ (electron) interacting with an Abelian gauge field $A_\mu(x)$ (photon) in n-dimensional Minkowski space with the metrics $x^2 \equiv x_\mu x_\mu = x_0^2 - x_1^2 - \cdots - x_{n-1}^2$. (For notation simplicity we write all vector indices as low ones) The

Lagrangian with a gauge fixing term has the form

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu} - \frac{1}{2d_l}(\partial_\mu A_\mu)^2 + \bar{\psi}(i\hat{\partial} - m + e\hat{A})\psi. \quad (1)$$

Here $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $\hat{A} \equiv A_\mu\gamma_\mu$, $\bar{\psi} = \psi^*\gamma_0$, m is an electron mass, e is a charge (coupling constant), d_l is a gauge parameter, γ_μ are Dirac matrices.

A generating functional of Green functions (vacuum expectations values for T-product of fields) can be represented as a functional integral

$$G(J,\eta) = \int D(\psi,\bar{\psi},A)\exp i\Big\{\int dx\Big(\mathcal{L} + J_\mu(x)A_\mu(x)\Big) - \int dxdy\bar{\psi}^\beta(y)\eta^{\beta\alpha}(y,x)\psi^\alpha(x)\Big\}. \quad (2)$$

Here $J_\mu(x)$ is a source of the gauge field, and $\eta^{\beta\alpha}(y,x)$ is a bilocal source of the spinor field (α and β are spinor indices). Normalization constant omitted.

Functional derivatives of G with respect to sources are vacuum expectation values

$$\frac{\delta G}{\delta J_\mu(x)} = i < 0 \mid A_\mu(x) \mid 0 >, \quad \frac{\delta G}{\delta\eta^{\beta\alpha}(y,x)} = i < 0 \mid T\Big\{\psi^\alpha(x)\bar{\psi}^\beta(y)\Big\} \mid 0 > . \quad (3)$$

A heuristic derivation of SDEs for the generating functional G is based on relations (see, for example, [6], [7])

$$0 = \int D(\psi,\bar{\psi},A)\frac{\delta}{\delta A_\mu(x)}\exp i\Big\{\int dx\Big(\mathcal{L} + J_\mu(x)A_\mu(x)\Big) - \int dxdy\bar{\psi}(y)\eta(y,x)\psi(x)\Big\}, \quad (4)$$

$$0 = \int D(\psi,\bar{\psi},A)\frac{\delta}{\delta\bar{\psi}(x)}\bar{\psi}(y)\exp i\Big\{\int dx\Big(\mathcal{L} + J_\mu(x)A_\mu(x)\Big) - \int dxdy\bar{\psi}(y)\eta(y,x)\psi(x)\Big\}. \quad (5)$$

Taking derivatives of eqs.(4)-(5) and taking into account eq.(3) we get SDEs for the generating functional of Green functions of QED

$$(g_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu + \frac{1}{d_l}\partial_\mu\partial_\nu)\frac{1}{i}\frac{\delta G}{\delta J_\nu(x)} + ie\,\mathrm{tr}\Big\{\gamma_\mu\frac{\delta G}{\delta\eta(x,x)}\Big\} + J_\mu(x)G = 0, \quad (6)$$

$$\delta(x-y)G + (i\hat{\partial} - m)\frac{\delta G}{\delta\eta(y,x)} + \frac{e}{i}\gamma_\mu\frac{\delta^2 G}{\delta J_\mu(x)\delta\eta(y,x)} - \int dx'\eta(x,x')\frac{\delta G}{\delta\eta(y,x')} = 0. \quad (7)$$

(Here and everywhere below ∂_μ denote a differentiation with respect to variable x, while a differentiation with respect to another variable will be denoted by indication of this variable as an upper index. For example, the differentiation in variable y will be denoted as ∂^y_μ.)

We consider in this and following Sections the unrenormalized theory, therefore for all singular divergent quantities some regularization is implied. A renormalization of SDEs is considered below (Section 3).

Gauge invariance imposes some limitations on solutions of SDEs (6) and (7) known as Ward identities. Acting by ∂_μ on eq.(6), we get a relation

$$\frac{1}{d_l}\partial^2\partial_\mu\frac{1}{i}\frac{\delta G}{\delta J_\mu(x)} + e\,\mathrm{tr}\Big\{i\hat{\partial}\frac{\delta G}{\delta\eta(x,x)}\Big\} + \partial_\mu J_\mu(x)G = 0, \quad (8)$$

which we shall name the first generating relation. One more relation follows from eq.(7) and its conjugated SDE

$$\delta(x-y)G + \frac{\delta G}{\delta\eta(y,x)}(-i\hat{\partial}^y - m) + \frac{e}{i}\frac{\delta^2 G}{\delta J_\mu(y)\delta\eta(y,x)}\gamma_\mu - \int dx'\frac{\delta G}{\delta\eta(x',x)}\eta(x',y) = 0. \quad (9)$$

Eq.(9) is a consequence of the relation

$$0 = \int D(\psi,\bar\psi,A)\frac{\delta}{\delta\psi(y)}\psi(x)\exp i\Big\{\int dx\Big(\mathcal{L}+J_\mu(x)A_\mu(x)\Big)-\int dxdy\bar\psi(y)\eta(y,x)\psi(x)\Big\}, \quad (10)$$

which is conjugated to (5). Subtract eq.(9) from eq.(7), take a trace over spinor indices and put $y = x$. As a result of these simple manipulations we get the relation

$$\mathrm{tr}\Big\{i\hat\partial\frac{\delta G}{\delta\eta(x,x)}\Big\} = \int dx'\,\mathrm{tr}\Big\{\eta(x,x')\frac{\delta G}{\delta\eta(x,x')}-\frac{\delta G}{\delta\eta(x',x)}\eta(x',x)\Big\}, \quad (11)$$

which we shall name the second generating relation. Relation (11) is fulfilled really for numerous class of interactions local with respect to fermions (see [8]). For QED we can combine both relation into one, substituting $\mathrm{tr}\Big\{i\hat\partial\frac{\delta G}{\delta\eta(x,x)}\Big\}$ from (11) into (8). As a result we get

$$\frac{i}{d_l}\partial^2\partial_\mu\frac{\delta G}{\delta J_\mu(x)} = \partial_\mu J_\mu(x)G + e\int dx'\,\mathrm{tr}\Big\{\eta(x,x')\frac{\delta G}{\delta\eta(x,x')}-\frac{\delta G}{\delta\eta(x',x)}\eta(x',x)\Big\}, \quad (12)$$

which is the generating relation of Ward identities for QED.

Differentiating eq.(12) with respect to J_λ and switching off the sources we obtain the known relation

$$\partial^2\partial_\mu D_{\mu\lambda}(x-y) = d_l\partial_\lambda\delta(x-y), \quad (13)$$

where

$$D_{\mu\lambda}(x-y) \equiv i\frac{\delta^2 G}{\delta J_\lambda(y)\delta J_\mu(x)}\Big|_{J=\eta=0} \quad (14)$$

is the photon propagator. From relation (13) it follows that the longitudinal part of the full photon propagator (in momentum space) is as follows:

$$D^{long}_{\mu\lambda}(k) = -d_l\frac{k_\mu k_\lambda}{(k^2)^2}. \quad (15)$$

Differentiating eq.(12) with respect to η, we obtain after switching off the sources another known relation

$$\frac{i}{d_l}\partial^2\partial_\mu F_\mu(x;x',y') = e[\delta(x-y')-\delta(x-x')]S(x'-y'). \quad (16)$$

Here

$$F_\mu(x;x',y') \equiv \frac{\delta^2 G}{\delta J_\mu(x)\delta\eta(y',x')}\Big|_{J=\eta=0} \quad (17)$$

is the three-point function, and

$$S(x-y) \equiv \frac{\delta G}{\delta\eta(y,x)}\Big|_{J=\eta=0} \quad (18)$$

is the full electron propagator.

Relation (16) looks much familiar if one go to the amputated three-pointer (vertex function) Γ_μ which is defined as

$$\Gamma_\mu(x\mid x',y') \equiv \int dx_1dx_1'dy_1'S^{-1}(x'-x_1')F_\nu(x_1;x_1',y_1')S^{-1}(y_1'-y')D^{-1}_{\nu\mu}(x_1-x). \quad (19)$$

Exploiting this definition and eq.(13), we obtain the Ward identity for vertex function

$$i\partial_\mu\Gamma_\mu(x\mid x',y') = e[\delta(x-x')-\delta(x-y')]S^{-1}(x'-y'). \quad (20)$$

2 Iteration scheme

At $e = 0$ SDEs (6) and (7) have a solution

$$G^{free} = \exp\Big\{\frac{1}{2i} J_\mu \star D^c_{\mu\nu} \star J_\nu + \mathrm{Tr}\log(1 + S^c \star \eta)\Big\},$$

where

$$D^c_{\mu\nu} = [g_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu + \frac{1}{d_l}\partial_\mu\partial_\nu]^{-1}, \quad S^c = (m - i\hat{\partial})^{-1}$$

are free field propagators, and the sign $\star$ denotes a multiplication in operator sense. (Also in operator sense we shall treat Tr, as distinct from tr, which corresponds to the trace over spinor indices.) The functional G^{free} is a generating functional of Green functions for free fields and is a base for an iteration scheme of perturbation theory in coupling constant e.

For solution of SDEs (6) and (7) we shall use another iteration scheme proposed in works [9], [10]. A general idea of this scheme is an approximation of functional-differential SDEs (6) and (7) by equations with "constant", i.e. independent of the sources J_μ and η, coefficient. Thus we approximate functional-differential SDEs near the point $J_\mu = 0,\ \eta = 0$. Since the objects of calculations are Green functions, i.e., derivatives of G in zero, such approximation seems to be quite natural. A circumstance of no small importance is a simplicity of the leading approximation of this scheme. Equations for all subsequent approximations also can be easy written. Green functions of every order of this scheme are defined as solutions of a closed system of equations. Technically this scheme is not much more complicated in comparison with the coupling constant perturbation theory, but in contrast to the last one, it maintains an information which is inaccessible in any finite order of the perturbation theory, i.e. it is a nonperturbative method.

In correspondence with aforesaid, we choose as leading approximation equations a system of functional-differential equations

$$(g_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu + \frac{1}{d_l}\partial_\mu\partial_\nu)\frac{1}{i}\frac{\delta G^{(0)}}{\delta J_\nu(x)} + ie\,\mathrm{tr}\Big\{\gamma_\mu \frac{\delta G^{(0)}}{\delta\eta(x,x)}\Big\} = 0, \tag{21}$$

$$\delta(x-y)G^{(0)} + (i\hat{\partial} - m)\frac{\delta G^{(0)}}{\delta\eta(y,x)} + \frac{e}{i}\gamma_\mu \frac{\delta^2 G^{(0)}}{\delta J_\mu(x)\delta\eta(y,x)} = 0. \tag{22}$$

A solution of leading approximation equations (21)-(22) is the functional

$$G^{(0)} = \exp\Big\{iV_\mu \star J_\mu + \mathrm{Tr}\, S^{(0)} \star \eta\Big\}, \tag{23}$$

where $V_\mu(x)$ and $S^{(0)}(x,y)$ satisfy equations

$$(g_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu + \frac{1}{d_l}\partial_\mu\partial_\nu)V_\nu(x) + ie\,\mathrm{tr}\Big\{\gamma_\mu S^{(0)}(x,x)\Big\} = 0 \tag{24}$$

$$\delta(x-y) + (i\hat{\partial} - m)S^{(0)}(x,y) + e\hat{V}(x)S^{(0)}(x,y) = 0. \tag{25}$$

By analogy with the theory of ordinary differential equation we shall call these equations as characteristic ones.

In correspondence with the choice of the leading approximation ith term of the iteration expansion of the generating functional

$$G = G^{(0)} + G^{(1)} + \cdots + G^{(i)} + \cdots \tag{26}$$

is a solution of iteration scheme equations

$$(g_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu + \frac{1}{d_l}\partial_\mu\partial_\nu)\frac{1}{i}\frac{\delta G^{(i)}}{\delta J_\nu(x)} + ie\,\mathrm{tr}\Big\{\gamma_\mu\frac{\delta G^{(i)}}{\delta\eta(x,x)}\Big\} = -J_\mu(x)G^{(i-1)}, \tag{27}$$

$$\delta(x-y)G^{(i)} + (i\hat{\partial} - m)\frac{\delta G^{(i)}}{\delta\eta(y,x)} + \frac{e}{i}\gamma_\mu\frac{\delta^2 G^{(i)}}{\delta J_\mu(x)\delta\eta(y,x)} = \int dx'\eta(x,x')\frac{\delta G^{(i-1)}}{\delta\eta(y,x')}. \tag{28}$$

A solution of eqs.(27)-(28) is looked for in the form

$$G^{(i)} = P^{(i)}G^{(0)}. \tag{29}$$

On taking into account characteristic equations (24)-(25) a system of equations for $P^{(i)}$ assumes the form

$$(g_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu + \frac{1}{d_l}\partial_\mu\partial_\nu)\frac{1}{i}\frac{\delta P^{(i)}}{\delta J_\nu(x)} + ie\,\mathrm{tr}\Big\{\gamma_\mu\frac{\delta P^{(i)}}{\delta\eta(x,x)}\Big\} = -J_\mu(x)P^{(i-1)}, \tag{30}$$

$$(i\hat{\partial} - m)\frac{\delta P^{(i)}}{\delta\eta(y,x)} + \frac{e}{i}\gamma_\mu\frac{\delta^2 P^{(i)}}{\delta J_\mu(x)\delta\eta(y,x)} + e\hat{V}(x)\frac{\delta P^{(i)}}{\delta\eta(y,x)} + \frac{e}{i}\gamma_\mu S^{(0)}(x,y)\frac{\delta P^{(i)}}{\delta J_\mu(x)}$$
$$= \int dx'\eta(x,x')\Big\{\frac{\delta P^{(i-1)}}{\delta\eta(y,x')} + S^{(0)}(x',y)P^{(i-1)}\Big\}. \tag{31}$$

Since $P^{(0)} \equiv 1$, it is evident that for any i the functional $P^{(i)}$ is a polynomial in functional variables J and η. This circumstance is very important since it means the system of equations for coefficient functions of this functional to be closed in any order of the iteration scheme.

This iteration scheme has no explicit small parameter. In some sense, the sources J and η play the role of such a parameter. Expansion (26) of the generating functional should be treated as an approximation of $G(J,\eta)$ near the point $J_\mu = 0,\ \eta = 0$. In essence, instead of the question about the small parameter, one should put a question about the convergence of the iteration series. Studying the convergence properties of the expansion for the quantum-field-theoretical case is obviously a very complicated problem, and we have no any rigorous statement concerning this question. Nevertheless, some qualitative reasons can be given in favor of the following supposition: the convergence of such a type expansion in any case is not worse than the convergence of the coupling constant perturbation series. The reason is following: being the simplest from the practical point of view the coupling constant perturbation series is the worst in the mathematical sense. The matter is that a small parameter (the coupling constant) is a multiplier at the highest derivative terms of the functional-differential SDEs for the generating functional. This means that the equation is a singularly perturbed one and, consequently, the coupling constant perturbation series is an asymptotic series at best. From the other hand, the proposed expansion is regular in the mathematical sense and, consequently, possesses the better convergence properties. (See also [10] for more details.) In addition, note that for a model "zero-dimensional" theory at $n = 0$, for which the functional integral becomes the ordinary ones, and SDEs — ordinary differential equations, such an expansion possesses very good convergence properties [10].

Generating relation (12) of Ward identities in the framework of given iteration scheme takes the form

$$\frac{i}{d_l}\partial^2\partial_\nu\frac{\delta G^{(i)}}{\delta J_\nu(x)} = \partial_\nu J_\nu(x)G^{(i-1)} + \int dx'\,\mathrm{tr}\Big\{\eta(x,x')\frac{\delta G^{(i-1)}}{\delta\eta(x,x')} - \frac{\delta G^{(i-1)}}{\delta\eta(x',x)}\eta(x',x)\Big\}, \tag{32}$$

Its corollaries like eqs. (14), (16) and (20) should be modified correspondingly. These Ward identities are very useful tools to control the gauge invariance. As we can see below, the gauge invariance requirements, which are expressed in the form of Ward identities, impose some strict limitations in given iteration scheme (see, for example, discussions after eq. (96) and in Section 5). First step calculations, which are considered in present article, demonstrate their compatibility with the gauge invariance (though some question arises for the solutions with chiral symmetry breaking, see Sections 6 and Conclusion). Surely, at the moment we can say nothing on this problem for higher steps of iterations.

Consider in more detail the leading approximation described by eqs. (23), (24) and (25). It is evident that for a conservation of Poincaré-invariance of the theory one should assume $V_\mu \equiv 0$. Then from eq.(24) it follows that

$$\mathrm{tr}\left\{\gamma_\mu S^{(0)}(x,x)\right\} = 0. \tag{33}$$

By definition (see (18)) $S^{(0)}$ is the electron propagator at the leading approximation, therefore it follows from above mentioned Poincaré-invariance that $S^{(0)}(x,y) = S^{(0)}(x-y)$, and

$$S^{(0)}(x,x) = S^{(0)}(0) = \int \frac{dp}{(2\pi)^n} S^{(0)}(p).$$

As was stated above, a regularization is always supposed for all such like expressions.

As these conditions being fulfilled the solution of eq.(25) is the free propagator

$$S^{(0)} = S^c = (m - i\hat{\partial})^{-1}.$$

For this solution condition (33) is equivalent to the requirement imposed on a regularization

$$\int dp \frac{p_\mu}{m^2 - p^2} = 0.$$

It is very difficult to imagine an invariant regularization for which this condition would not be fulfilled.

So, the generating functional of leading approximation is

$$G^{(0)} = \exp\left\{\mathrm{Tr}\, S \star \eta\right\}. \tag{34}$$

As follows from eq.(34), the unique connected Green function of the leading approximation is the electron propagator. Other connected Green functions appear at following iteration steps. It is also follows from eq.(34) that the generating functional at leading approximation does not possess the complete Fermi-symmetry. Really, as follows from the definition of generating functional, the Fermi-symmetry implies on *full* generating functional the condition

$$\frac{\delta^2 G}{\delta\eta^{\beta\alpha}(y,x)\delta\eta^{\beta'\alpha'}(y',x')} = -\frac{\delta^2 G}{\delta\eta^{\beta'\alpha}(y',x)\delta\eta^{\beta\alpha'}(y,x')}. \tag{35}$$

Evidently condition (35) does not fulfilled for $G^{(0)}$ defined by eq.(34). The violation of this condition leads particularly to the violation of connected structure of the leading approximation two-particle (four-point) electron function which is

$$S_2^{(0)}(x,y;x',y') \equiv \left.\frac{\delta^2 G^{(0)}}{\delta\eta^{\beta\alpha}(y,x)\delta\eta^{\beta'\alpha'}(y',x')}\right|_{J=\eta=0} = S^c(x-y)S^c(x'-y') \tag{36}$$

– a term $-S^c(x-y')S^c(x'-y)$ is missed.

Such a situation is rather typical for nonperturbative calculational schemes with bilocal source (for example, for $1/N$-expansion in the bilocal source formalism), but discrepancy of such type are not an obstacle for using these iteration schemes. Really, condition (35) should be satisfied by the *full* generating functional G which is an *exact* solution of SDEs (6) and (7). It is clear that an approximate solution may do not possess all properties of an exact one. In given case we have just the same situation. Properties of connectivity and Fermi-symmetry of higher Green functions, which are not fulfilled at first steps of the iteration scheme, restore at subsequent steps. For example, the structure of disconnected part of the two-electron function is restored as early as at the first step of the iteration scheme (see below). At subsequent steps the correct connected structure of many-electron functions and other corollaries of Fermi-symmetry are reconstructed. Such stepwise reconstruction of exact solution properties is very natural for the given iteration scheme as it is based on an approximation of the generating functional $G(J,\eta)$ in vicinity of zero. The Green functions are coefficients of the generating functional expansion in the vicinity of zero, therefore only the lowest functions are well-described at first steps of the approximation – at the leading approximation the electron propagator only. Higher many-particle functions come into the play later, at following steps, and relation (35) is fulfilled more and more exactly when we go toward exact solution.

A solution of the first step equation is looked for in the form

$$G^{(1)} = P^{(1)} G^{(0)},$$

where polynomial $P^{(1)}$ is a solution of eqs.(30) and (31) at $i=1$ and $V_\mu \equiv 0$. This solution is

$$P^{(1)} = \frac{1}{2}\eta \star S_2^{(1)} \star \eta + S^{(1)} \star \eta + \frac{1}{2i} J_\mu \star D_{\mu\nu}^{(1)} \star J_\nu + J_\mu \star F_\mu^{(1)} \star \eta + i\mathcal{A}_\mu^{(1)} \star J_\mu. \tag{37}$$

In correspondence with above definitions, $S_2^{(1)}(x,y;x',y')$ is a two-electron function, $S^{(1)}(x-y)$ is a correction to the electron propagator, $D_{\mu\nu}^{(1)}(x-y)$ is a photon propagator, $F_\mu^{(1)}(x;x',y')$ is a three-point function, $\mathcal{A}_\mu^{(1)}$ is a photon-field vacuum expectation value. Upper index denotes the first-step iteration scheme quantities.

Equations for first-step functions follow from eqs.(30),(31) at $i=1$, $V_\mu \equiv 0$ and they have the form

$$(g_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu + \frac{1}{d_l}\partial_\mu\partial_\nu) D_{\nu\lambda}^{(1)}(x-y) - ie \,\mathrm{tr}\left\{\gamma_\mu F_\lambda^{(1)}(y \mid x,x)\right\} = g_{\mu\lambda}\delta(x-y), \tag{38}$$

$$(g_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu + \frac{1}{d_l}\partial_\mu\partial_\nu) F_\nu^{(1)}(x \mid x',y') = e \,\mathrm{tr}\left\{\gamma_\mu S_2^{(1)}(x,x;x',y')\right\}, \tag{39}$$

$$(g_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu + \frac{1}{d_l}\partial_\mu\partial_\nu) \mathcal{A}_\nu^{(1)}(x) + ie \,\mathrm{tr}\left\{\gamma_\mu S^{(1)}(x,x)\right\} = 0, \tag{40}$$

$$(i\hat{\partial} - m) S_2^{(1)}(x,y;x',y') - ie\gamma_\mu S^c(x-y) F_\mu^{(1)}(x \mid x',y') = \delta(x-y') S^c(x'-y), \tag{41}$$

$$(i\hat{\partial} - m) F_\lambda^{(1)}(z \mid x,y) = e\gamma_\mu S^c(x-y) D_{\mu\lambda}^{(1)}(x-z), \tag{42}$$

$$(i\hat{\partial} - m) S^{(1)}(x-y) - ie\gamma_\mu F_\mu^{(1)}(x \mid x,y) + e\gamma_\mu S^c(x-y)\mathcal{A}_\mu^{(1)}(x) = 0. \tag{43}$$

The system of eqs.(38)-(43) at first sight seems to be overfilled: six equations for five functions. Really one can prove that eq.(39) is a corollary of other equations. Further, if one imposes on $S^{(1)}$ the same condition as for $S^{(0)}$ (see (33)), then the existence of trivial solution $\mathcal{A}^{(1)}_\mu \equiv 0$ for $\mathcal{A}^{(1)}_\mu$ follows from eq.(40). We restrict ourselves to this solution for $\mathcal{A}^{(1)}_\mu$ ignoring all others as violating Poincaré-invariance of the theory.

From eq.(42) we obtain

$$F^{(1)}_\lambda(z \mid x, y) = -e \int dx' S^c(x - x')\gamma_\mu S^c(x' - y) D^{(1)}_{\mu\lambda}(x' - z). \tag{44}$$

Substituting (44) into (38), we obtain after simple transformations

$$(D^{(1)}_{\mu\nu})^{-1} = (D^c_{\mu\nu})^{-1} + \Pi_{\mu\nu} \tag{45}$$

where

$$\Pi_{\mu\nu}(x) = ie^2 \,\mathrm{tr}\left\{\gamma_\mu S^c(x)\gamma_\nu S^c(-x)\right\} \tag{46}$$

is a free-electron loop. It follows from Ward identities that $\Pi_{\mu\nu}$ is transversal in the momentum space:

$$\Pi_{\mu\nu}(k) = \Pi(k^2)\,\pi_{\mu\nu}, \tag{47}$$

where

$$\pi_{\mu\nu} \equiv g_{\mu\nu} - \frac{k_\mu k\nu}{k^2}$$

is transverse projector. On taking into account eq. (47) we finally obtain (in the momentum space)

$$D^{(1)}_{\mu\nu}(k) = \frac{1}{-k^2 + \Pi(k^2)}\,\pi_{\mu\nu} - d_l \frac{k_\mu k_\nu}{(k^2)^2}. \tag{48}$$

Then we obtain from eq. (41) a two-electron function

$$S^{(1)}_2(x, y; x', y') = -S^c(x - y')S^c(x' - y) + \\ + ie^2 \int dx_1 dx_2 S^c(x - x_1)\gamma_\mu S^c(x_1 - y) D^{(1)}_{\mu\nu}(x_1 - x_2) S^c(x' - x_2)\gamma_\nu S^c(x_2 - y'), \tag{49}$$

and from eq. (43)– a correction to the electron propagator

$$S^{(1)}(x - y) = ie^2 \int dx_1 dx_2 S^c(x - x_1)\gamma_\mu S^c(x_1 - x_2) D^{(1)}_{\mu\nu}(x_1 - x_2)\gamma_\nu S^c(x_2 - y). \tag{50}$$

Thus we have obtained the expressions for all first-step functions. Note, that a disconnected part of function $S^{(1)}_2$ (see (49)) is the "missing" disconnected part of the leading-approximation two-electron function (36). Hence, as was mentioned earlier, the correct structure of the disconnected part of the two-electron function is reconstructed at the first step of the calculations. At subsequent steps a correct crossing-symmetrical structure of the connected part is also reconstructed.

Before renormalization the formulae obtained are formal expressions due to ultraviolet divergences. The question arises: is there an explicit regularization scheme, which is consistent with the considered iterative expansion? Though in general case we have no positive answer for this question for any finite step of iteration, at the same time we have no reasons to suppose the usual well-known regularization schemes do not work in our case. In particular, the ultraviolet divergence of the free-electron loop (46) can be treated with dimensional

regularization. Note, that at $n = 2$ (two-dimensional electrodynamics) this loop converges in dimensional regularization. At $m = 0$ (Schwinger model)

$$\Pi(k^2) = \frac{e^2}{\pi}, \tag{51}$$

and the photon propagator (in transverse gauge) is

$$D^{(1)}_{\mu\nu}(k) = \frac{1}{\frac{e^2}{\pi} - k^2}\,\pi_{\mu\nu}, \tag{52}$$

which coincides with the exact result [11].

3 Renormalization

So far we have considered unrenormalized theory. To give a physical sense to quantities to be calculated in a theory with ultraviolet divergences it is necessary to carry out the renormalization procedure.

The Lagrangian of QED for the renormalized fields has the form

$$\mathcal{L}_r = -\frac{Z_3}{4}F_{\mu\nu}F_{\mu\nu} - \frac{1}{2d_l}(\partial_\mu A_\mu)^2 + Z_1\bar\psi(i\hat\partial - m_r + e_r\hat A)\psi - \delta m\bar\psi\psi. \tag{53}$$

Here ψ and A_μ are the renormalized fields, Z_1 and Z_3 are renormalization constants of spinor and gauge fields correspondingly, m_r and e_r are renormalized mass and charge, δm is the counterterm of electron-mass renormalization. We have taken into account the fact, that due to the gauge invariance and Ward identities the renormalization constants of the spinor field and of the interaction are equal: $Z_1 = Z_2$, and the longitudinal part of the gauge field is not renormalized.

SDEs for a generating functional of renormalized Green functions are

$$[Z_3(g_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu) + \frac{1}{d_l}\partial_\mu\partial_\nu]\frac{1}{i}\frac{\delta G}{\delta J_\nu(x)} + ie_rZ_1\,\mathrm{tr}\left\{\gamma_\mu\frac{\delta G}{\delta\eta(x,x)}\right\} + J_\mu(x)G = 0, \tag{54}$$

$$\delta(x-y)G + Z_1(i\hat\partial - m_r)\frac{\delta G}{\delta\eta(y,x)} - \delta m\frac{\delta G}{\delta\eta(y,x)} + \frac{e_rZ_1}{i}\gamma_\mu\frac{\delta^2 G}{\delta J_\mu(x)\delta\eta(y,x)} = \\ = \int dx'\eta(x,x')\frac{\delta G}{\delta\eta(y,x')}. \tag{55}$$

Generating relation (12) of Ward identities for the renormalized generating functional preserves its form under substitution $e \to e_r$. Correspondingly, its corollaries (13), (16) and (20) are preserved with the following distinction: all quantities are renormalized ones.

Being applied to our iteration scheme, the renormalization procedure means: the generating functional G as well as all renormalization constants and counterterms has corresponding iteration expansions:

$$Z_j = Z_j^{(0)} + Z_j^{(1)} + \cdots, \quad \delta m = \delta m^{(0)} + \delta m^{(1)} + \cdots$$

Thus renormalized equations of leading approximation are

$$[Z_3^{(0)}(g_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu) + \frac{1}{d_l}\partial_\mu\partial_\nu]\frac{1}{i}\frac{\delta G^{(0)}}{\delta J_\nu(x)} + ie_r Z_1^{(0)}\,\mathrm{tr}\left\{\gamma_\mu\frac{\delta G^{(0)}}{\delta\eta(x,x)}\right\} = 0, \tag{56}$$

$$\delta(x-y)G^{(0)} + Z_1^{(0)}(i\hat{\partial} - m_r)\frac{\delta G^{(0)}}{\delta\eta(y,x)} - \delta m^{(0)}\frac{\delta G^{(0)}}{\delta\eta(y,x)} + \frac{e_r Z_1^{(0)}}{i}\gamma_\mu\frac{\delta^2 G^{(0)}}{\delta J_\mu(x)\delta\eta(y,x)} = 0. \tag{57}$$

A solution of these equations is the same leading approximation functional (34) but now S^c is the renormalized free propagator

$$S^c = (m_r - i\hat{\partial})^{-1}.$$

At that

$$Z_1^{(0)} = 1, \quad \delta m^{(0)} = 0,$$

and the same condition (33) is imposed for a regularization. Note that the value of constant $Z_3^{(0)}$ remains undefined in the framework of leading approximation. This constant is fixed at the first step of iteration scheme when the photon propagator comes into play.

The first-step functional is $G^{(1)} = P^{(1)}G^{(0)}$, and taking into account above definitions and results of the leading approximations we obtain for $P^{(1)}$ the equations

$$[Z_3^{(0)}(g_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu) + \frac{1}{d_l}\partial_\mu\partial_\nu]\frac{1}{i}\frac{\delta P^{(1)}}{\delta J_\nu(x)} + ie_r\,\mathrm{tr}\left\{\gamma_\mu\frac{\delta P^{(1)}}{\delta\eta(x,x)}\right\} = -J_\mu(x), \tag{58}$$

$$(i\hat{\partial} - m_r)\frac{\delta P^{(1)}}{\delta\eta(y,x)} + \frac{e_r}{i}\gamma_\mu\frac{\delta^2 P^{(1)}}{\delta J_\mu(x)\delta\eta(y,x)} + \frac{e_r}{i}\gamma_\mu S^c(x-y)\frac{\delta P^{(1)}}{\delta J_\mu(x)} =$$
$$= \int dx_1\eta(x,x_1)S^c(x_1-y) + Z_1^{(1)}\delta(x-y) + \delta m^{(1)}S^c(x-y). \tag{59}$$

A solution of these equations has the same form (37) with a distinction: the coefficient functions of polynomial (37), which define first-step Green functions, are now the renormalized quantities. We do not write out equations for the coefficient functions which coincides with above unrenormalized equations (38)-(43) up to evident from (58) and (59) modifications.

For three-pointer $F_\mu^{(1)}$ we obtain the same expression (44), and for the photon propagator in the momentum space we have

$$D_{\mu\nu}^{(1)}(k) = \frac{1}{-Z_3^{(0)}k^2 + \Pi(k^2)}\,\pi_{\mu\nu} - d_l\frac{k_\mu k_\nu}{(k^2)^2}, \tag{60}$$

where Π is the same free-electron loop (46)-(47) with a substitution $e \to e_r$ $m \to m_r$.

Expression (49) for two-electron function $S_2^{(1)}$ is not changed (with the same substitution again). For the first-step correction to the electron propagator we have

$$S^{(1)}(x-y) = ie_r^2\int dx_1dx_2S^c(x-x_1)\gamma_\mu S^c(x_1-x_2)D_{\mu\nu}^{(1)}(x_1-x_2)\gamma_\nu S^c(x_2-y)$$
$$+Z_1^{(1)}S^c(x-y) - \delta m^{(1)}\int dx_1S^c(x-x_1)S^c(x_1-y). \tag{61}$$

If photon propagator (60) possesses a pole in the point $k^2 = \mu^2$ which corresponds to a particle with mass μ, then for $\Pi(k^2)$ the following normalization conditions should be satisfied

$$\Pi(\mu^2) = Z_3^{(0)}\mu^2, \quad \Pi'(\mu^2) = Z_3^{(0)} - 1. \tag{62}$$

In two-dimensional case ($n = 2$)

$$\Pi(k^2) = -\frac{e_r^2 k^2}{\pi}\int_0^1 dx \frac{x(1-x)}{m_r^2 - x(1-x)k^2 - i0}. \tag{63}$$

In particular at $m_r = 0$: $\Pi(k^2) = e_r^2/\pi$, and normalization conditions (62) give $Z_3^{(0)} = 1$, $\mu^2 = e_r^2/\pi$, i.e. the result for Schwinger model is unchanged by the renormalization.

For four-dimensional theory $\Pi(k^2)$ is an ultraviolet-divergent quantity. At the dimensional regularization ($n = 4 - 2\epsilon$)

$$\Pi(k^2) = -\frac{e_r^2 k^2}{2\pi^2}(2\pi)^\epsilon \Gamma(\epsilon)\int_0^1 dx \frac{x(1-x)}{(m_r^2 - x(1-x)k^2 - i0)^\epsilon}. \tag{64}$$

Here $\Gamma(x)$ is gamma-function. In this case the normalization conditions give us the following

$$Z_3^{(0)} = 1 - \frac{\alpha_r}{3\pi}\Big\{\frac{1}{\epsilon} + \psi(1) + \log 2\pi - \log\frac{m_r^2}{M^2}\Big\}, \quad \mu^2 = 0.$$

Here $\alpha_r = e_r^2/4\pi$, $\psi(1)$ is Euler constant, M^2 is a mass parameter of the dimensional regularization.

In the Euclidean region $k^2 < 0$ the renormalized photon propagator in four-dimensional theory possesses a non-physical pole at $k^2 \simeq -m_r^2 \exp\{\frac{3\pi}{\alpha_r}\}$ – it is Landau pole [12]. This pole arises also at renormalization-group summation [1], and in the framework of $1/N$-expansion [13]. A presence of this pole is a serious difficulty and, in particular, prevents one from making sensible calculations (or needs some refined construction which seems to be superfluous for QED [3]). In the coupling constant perturbation theory one can exploit a smallness of α_r to avoid the troubles, but in used nonperturbative scheme the unique non-contradictory possibility is the choice $\alpha_r = 0$, and the theory becomes trivial.

Therefore, as a result of investigation of the first-step equation of our calculational scheme the following conclusion is made: for the four-dimensional QED the renormalization of theory leads to triviality as early as at the first step of calculations. This result, from one side, demonstrates that proposed scheme correctly catches a nonperturbative content of QED, but, from the other side, such a version of the scheme is practically useless in contrast to the usual perturbation theory. However, the other version of the same calculational scheme exists which permits one to make sensible nonperturbative calculations in QED. To state this version is a matter of the remainder of the paper.

4 Nonperturbative vacuum

The iteration scheme considered above can be modified in such a manner that it becomes insensitive to triviality at least for first nontrivial steps of the calculations, i.e. applicable to nonperturbative calculations in QED. It means a possibility to calculate physical quantities without contradiction with the triviality of theory, in a similar way it is made in the

perturbation theory in renormalized coupling constant. Of course, one cannot say that the situation is fully analogous with coupling constant perturbation theory. In the framework of the perturbation theory one can be sure that calculations in any finite order do not lead to pathologies. It would be untimely to allege the same statement for the given iteration scheme. One cannot exclude at the moment the possibility of a pathology (which originates from the triviality of four-dimensional QED) in a higher step of the iterations. In the case of such pathology, the use of a grand unification for its elimination seems to be inevitable. A full answer on these principal question can be given only with detail investigations of higher steps of iterations. Nevertheless, as we shall see, an analysis of first steps of given iteration scheme demonstrates that it includes in a natural way such well-known nonperturbative treatments of QED as ladder and rainbow approximations.

To pass this modification of the iteration scheme (which we shall name calculations over nonperturbative vacuum) let us resolve SDE (6) with regard to the first derivative of the generating functional with respect J_μ:

$$\frac{1}{i}\frac{\delta G}{\delta J_\mu(x)} = -\int dx_1 D^c_{\mu\nu}(x-x_1)\Big\{J_\nu(x_1)G + ie\,\mathrm{tr}\,\gamma_\nu\frac{\delta G}{\delta\eta(x_1,x_1)}\Big\}, \tag{65}$$

and substitute it into the second SDE (7). As a result we obtain the "integrated over A_μ" (in the functional-integral terminology) equation

$$\delta(x-y)G + (i\hat\partial - m)\frac{\delta G}{\delta\eta(y,x)} + \frac{e^2}{i}\int dx_1 D^c_{\mu\nu}(x-x_1)\gamma_\mu\frac{\delta}{\delta\eta(y,x)}\,\mathrm{tr}\,\gamma_\nu\frac{\delta G}{\delta\eta(x_1,x_1)} =$$
$$= \int dx_1\Big\{\eta(x,x_1)\frac{\delta G}{\delta\eta(y,x_1)} + eD^c_{\mu\nu}(x-x_1)J_\nu(x_1)\gamma_\mu\frac{\delta G}{\delta\eta(y,x)}\Big\}. \tag{66}$$

Exploiting Fermi-symmetry condition (35) let us transform eq.(66) in following manner:

$$\delta(x-y)G + (i\hat\partial - m)\frac{\delta G}{\delta\eta(y,x)} + ie^2\int dx_1 D^c_{\mu\nu}(x-x_1)\gamma_\mu\frac{\delta}{\delta\eta(x_1,x)}\gamma_\nu\frac{\delta G}{\delta\eta(y,x_1)} =$$
$$= \int dx_1\Big\{\eta(x,x_1)\frac{\delta G}{\delta\eta(y,x_1)} + eD^c_{\mu\nu}(x-x_1)J_\nu(x_1)\gamma_\mu\frac{\delta G}{\delta\eta(y,x)}\Big\}. \tag{67}$$

From the point of view of *exact* solutions equations (66) and (67) are fully equivalent since the transition from eq.(66) to eq.(67) is, in essence, an identical transformation. However, it is not the case for the used iteration scheme since, as was said earlier, condition (35) is fulfilled only approximately at any finite step of the iteration scheme. Therefore, eqs. (66) and (67) lead to different expansions. Eq.(66) leads, in essence, to the same calculational scheme as above, but eq.(67) gives us a fruitful scheme of calculation of physical quantities.

Before the writing equations of the scheme note that the photon propagator is not contained directly in coefficient functions of polynomials $P^{(i)}$ of the iteration scheme for eqs. (66) and (67) since these equations are, as pointed above, the result of "integration" over the photon variable A_μ. For calculation of the photon propagator and concerning quantities one should exploit the Dyson formula

$$D_{\mu\nu}(x-y) \equiv i\frac{\delta^2 G}{\delta J_\nu(y)\delta J_\mu(x)}\Big|_{J=\eta=0} = D^c_{\mu\nu}(x-y) + ie\int dx_1 D^c_{\mu\rho}(x-x_1)\,\mathrm{tr}\,\gamma_\rho F_\nu(y;x_1,x_1), \tag{68}$$

which is a consequence of a single differentiation of eq.(65).

In correspondence with the general principle of iteration construction we choose as the leading approximation eqs. (66) and (67) with $J_\mu = 0$, $\eta = 0$ in their coefficients. These equations are

$$\delta(x-y)G^{(0)} + (i\hat{\partial} - m)\frac{\delta G^{(0)}}{\delta\eta(y,x)} + \frac{e^2}{i}\int dx_1 D^c_{\mu\nu}(x-x_1)\gamma_\mu \frac{\delta}{\delta\eta(y,x)}\,\mathrm{tr}\,\gamma_\nu \frac{\delta G^{(0)}}{\delta\eta(x_1,x_1)} = 0 \quad (69)$$

and

$$\delta(x-y)G^{(0)} + (i\hat{\partial} - m)\frac{\delta G^{(0)}}{\delta\eta(y,x)} + ie^2\int dx_1 D^c_{\mu\nu}(x-x_1)\gamma_\mu \frac{\delta}{\delta\eta(x_1,x)}\gamma_\nu \frac{\delta G^{(0)}}{\delta\eta(y,x_1)} = 0. \quad (70)$$

The functional

$$G^{(0)} = \exp\Big\{\,\mathrm{Tr}\,S^{(0)} \star \eta\Big\}, \quad (71)$$

is a solution for both these equations, but, whereas the solution of the characteristic equation corresponding to eq.(69) is the free propagator $S^{(0)} = S^c$ with condition (33), a characteristic equation corresponding to eq.(70) has the form

$$[S^{(0)}]^{-1}(x) = (m - i\hat{\partial})\delta(x) - ie^2 D^c_{\mu\nu}(x)\gamma_\mu S^{(0)}(x)\gamma_\nu, \quad (72)$$

i.e., it is a non-trivial nonlinear equation for $S^{(0)}$, and so we name this scheme the calculations over nonperturbative vacuum as distinct from above scheme of eq.(66) which is based on the free solution S^c.

A detailed discussion of eq.(72) is given below. At the moment we note that at $m = 0$ (chiral limit) in transverse gauge $d_l = 0$ this equation possesses a simple solution

$$S^{(0)} = -1/i\hat{\partial}. \quad (73)$$

Really, in coordinate space[2] at $n > 2$

$$D^c_{\mu\nu}(x) = \frac{e^{-i\pi n/2}\Gamma(\frac{n}{2}-1)}{4i\pi^{n/2}(x^2 - i0)^{n/2-1}}\Big[\frac{1+d_l}{2}g_{\mu\nu} + (1-d_l)(\frac{n}{2}-1)\frac{x_\mu x_\nu}{x^2 - i0}\Big]. \quad (74)$$

In transverse gauge $d_l = 0$ the function $D^c_{\mu\nu}(x)$ possesses an important property ("$\hat{x}$-transversality")

$$D^c_{\mu\nu}(x)\gamma_\mu \hat{x}\gamma_\nu = 0. \quad (75)$$

From this property the existence of solution (73) at $m = 0$ follows immediately.

At $m \neq 0$ solving of the characteristic equation is rather complicated problem which needs application of some approximate or numerical methods. The search of solutions, breaking the chiral symmetry, is also of great interest (see below, Section 6).

An iteration equation for the nonperturbative vacuum in correspondence with eqs.(67) and (70) is

$$\delta(x-y)G^{(i)} + (i\hat{\partial} - m)\frac{\delta G^{(i)}}{\delta\eta(y,x)} + ie^2\int dx_1 D^c_{\mu\nu}(x-x_1)\gamma_\mu \frac{\delta}{\delta\eta(x_1,x)}\gamma_\nu \frac{\delta G^{(i)}}{\delta\eta(y,x_1)} =$$
$$= \int dx_1\Big\{\eta(x,x_1)\frac{\delta G^{(i-1)}}{\delta\eta(y,x_1)} + eD^c_{\mu\nu}(x-x_1)J_\nu(x_1)\gamma_\mu \frac{\delta G^{(i-1)}}{\delta\eta(y,x)}\Big\}. \quad (76)$$

[2]The case $n = 2$ is discussed below, see Conclusion

A solution of the first-step equation is $G^{(1)} = P^{(1)}G^{(0)}$, where

$$P^{(1)} = \frac{1}{2}\eta \star S_2^{(1)} \star \eta + S^{(1)} \star \eta + J_\mu \star F_\mu^{(1)} \star \eta. \tag{77}$$

With taking into account leading approximation (70) and characteristic equation (72) we obtain for three-point function $F_\lambda^{(1)}$, two-electron function $S_2^{(1)}$ and propagator correction $S^{(1)}$ the following equations

$$F_\lambda^{(1)}(z;x,y) = -e\int dx_1 D^c_{\lambda\mu}(z-x_1)S^{(0)}(x-x_1)\gamma_\mu S^{(0)}(x_1-y) + \\ +ie^2\int dx_1 dy_1 D^c_{\mu\nu}(x_1-y_1)S^{(0)}(x-x_1)\gamma_\mu F_\lambda^{(1)}(z;x_1,y_1)\gamma_\nu S^{(0)}(y_1-y), \tag{78}$$

$$S_2^{(1)}(x,y;x',y') = -S^{(0)}(x-y')S^{(0)}(x'-y) + \\ +ie^2\int dx_1 dy_1 D^c_{\mu\nu}(x_1-y_1)S^{(0)}(x-x_1)\gamma_\mu S_2^{(1)}(x_1,y_1;x',y')\gamma_\nu S^{(0)}(y_1-y), \tag{79}$$

$$S^{(1)}(x-y) = ie^2\int dx_1 dy_1 D^c_{\mu\nu}(x_1-y_1)S^{(0)}(x-x_1)\gamma_\mu S_2^{(1)}(x_1,y_1;y_1,y)\gamma_\nu + \\ +ie^2\int dx_1 dy_1 D^c_{\mu\nu}(x_1-y_1)S^{(0)}(x-x_1)\gamma_\mu S^{(1)}(x_1-y_1)\gamma_\nu S^{(0)}(y_1-y). \tag{80}$$

The first-step equations (78)–(80) are much more complicated in comparison with the first-step equations over perturbative vacuum considered above. In diagram language perturbative-vacuum equations (38)–(43) correspond to the summation of "chains", and its solutions can be easily written out in a general form (see (44)–(50)). Equations (78)–(80) and characteristic equation (72) in the diagram language correspond to the well-known ladder approximation. Such type equations for separate Green functions repeatedly were written out and investigated in literature (see [14]–[16], and also [17] and references therein) as the simplest nonperturbative approximation for exact Dyson equations [18] which constitute an infinite system of engaging equations. In our treatment, in contrast to preceding investigations, these equation are not a result of more or less arbitrary truncation of Dyson equations, but are a consistent part of the iteration scheme. This circumstance is very important for solving such problems as taking into account requirements of gauge invariance and renormalizability, which often became stumbling-blocks at investigation of nonperturbative approximations. So, for example, one can often encounter the statement that equation (72) for electron propagator contradicts Ward identity (16) and, consequently is not consistent with gauge invariance (see discussion of this problem in [17]). However, one should not forget that a comparison of an *approximate* equation (72) with the *exact* Ward identity (16) is not correct, and when an iteration scheme is absent the formulation of this problem, strongly speaking, is incorrect itself. In the framework of our iteration scheme this problem is solved very simply. As follows from eq. (32), Ward identity $F_\mu^{(i)}$ in this framework has the form

$$\frac{i}{d_l}\partial^2\partial_\nu F_\nu^{(i)}(x;x',y') = e[\delta(x-y') - \delta(x-x')]S^{(i-1)}(x'-y'). \tag{81}$$

It is easy to prove that for $i = 1$ relation (81) and equation (72) are consistent with the first-step equation (78) for $F_\mu^{(1)}$, i.e. the requirement of gauge invariance is fulfilled.

Turn to the renormalization. A renormalized equation for $\delta G/\delta J_\mu$ has the form

$$\frac{1}{i}\frac{\delta G}{\delta J_\mu(x)} = -\int dx_1 D^c_{\mu\nu}(x-x_1 \mid Z_3)\Big\{J_\nu(x_1)G + Z_1 i e_r \,\mathrm{tr}\,\gamma_\nu \frac{\delta G}{\delta\eta(x_1,x_1)}\Big\} \tag{82}$$

where

$$D^c_{\mu\nu}(Z_3) = [Z_3(g_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu) + \frac{1}{d_l}\partial_\mu\partial_\nu]^{-1}.$$

Correspondingly, renormalized SDE (67) is

$$\begin{aligned}
\delta(x-y)G + Z_1(i\hat\partial - m_r)\frac{\delta G}{\delta\eta(y,x)} - \delta m\frac{\delta G}{\delta\eta(y,x)} + \\
+ i(e_r Z_1)^2\int dx_1 D^c_{\mu\nu}(x-x_1 \mid Z_3)\gamma_\mu\frac{\delta}{\delta\eta(x_1,x)}\gamma_\nu\frac{\delta G}{\delta\eta(y,x_1)} = \\
= \int dx_1\Big\{\eta(x,x_1)\frac{\delta G}{\delta\eta(y,x_1)} + Z_1 e D^c_{\mu\nu}(x-x_1\mid Z_3)J_\nu(x_1)\gamma_\mu\frac{\delta G}{\delta\eta(y,x)}\Big\}.
\end{aligned} \tag{83}$$

A renormalized leading-approximation equation is

$$\begin{aligned}
\delta(x-y)G^{(0)} + Z_1^{(0)}(i\hat\partial - m_r)\frac{\delta G^{(0)}}{\delta\eta(y,x)} - \delta m^{(0)}\frac{\delta G^{(0)}}{\delta\eta(y,x)} + \\
+ i(e_r Z_1^{(0)})^2\int dx_1 D^c_{\mu\nu}(x-x_1\mid Z_3^{(0)})\gamma_\mu\frac{\delta}{\delta\eta(x_1,x)}\gamma_\nu\frac{\delta G^{(0)}}{\delta\eta(y,x_1)} = 0.
\end{aligned} \tag{84}$$

Equation (84) has a solution in the form of the same functional (71), where $S^{(0)}$ is a solution of renormalized characteristic equation

$$[S^{(0)}]^{-1}(x) = (Z_1^{(0)}(m_r - i\hat\partial) + \delta m^{(0)})\delta(x) - i(e_r Z_1^{(0)})^2 D^c_{\mu\nu}(x\mid Z_3^{(0)})\gamma_\mu S^{(0)}(x)\gamma_\nu. \tag{85}$$

A renormalized photon propagator is defined by Dyson formula

$$D_{\mu\nu}(x-y) = D^c_{\mu\nu}(x-y\mid Z_3) + ie_r Z_1\int dx_1 D^c_{\mu\rho}(x-x_1\mid Z_3)\,\mathrm{tr}\,\gamma_\rho F_\nu(y;x_1,x_1). \tag{86}$$

Since $F_\nu^{(0)}\equiv 0$ at the leading approximation, then

$$D^{(0)}_{\mu\nu} = D^c_{\mu\nu}(Z_3^{(0)}) = \frac{1}{Z_3^{(0)}\partial^2}(g_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\partial^2}) + d_l\frac{\partial_\mu\partial_\nu}{(\partial^2)^2}.$$

From the normalization condition for the photon propagator we obtain

$$Z_3^{(0)} = 1,$$

therefore, in all above formulae one can replace $D^c_{\mu\nu}(Z_3^{(0)})$ with $D^c_{\mu\nu}$.

In general case the renormalized photon propagator in the momentum space is

$$D_{\mu\nu}(k) = D^c_{\mu\nu}(k\mid Z_3) + ie_r Z_1 D^c_{\mu\rho}(k\mid Z_3)\int\frac{dp}{(2\pi)^n}\,\mathrm{tr}\,\Big\{\gamma_\rho F_\nu(k;p)\Big\},$$

where $F_\nu(k;p)$ is a Fourier-image of the three-pointer:

$$F_\nu(z;x,y) = \int\frac{dp}{(2\pi)^n}\frac{dq}{(2\pi)^n}\frac{dk}{(2\pi)^n}e^{-ipx+iqy-ikz}(2\pi)^n\delta(p-q+k)F_\nu(k;p). \tag{87}$$

The transversality follows from Ward identities:

$$ie_r Z_1 \int \frac{dp}{(2\pi)^n} \operatorname{tr}\left\{\gamma_\rho F_\nu(k;p)\right\} = \pi_{\rho\nu} f(k^2)$$

and, taking into account given definition of function $f(k^2)$, we obtain

$$D_{\mu\nu}(k) = -\frac{1+f(k^2)}{Z_3 k^2}\pi_{\mu\nu} - d_l \frac{k_\mu k_\nu}{(k^2)^2}. \tag{88}$$

Zero mass normalization condition ("photon normalization ") gives us the following

$$Z_3 = 1 + f(0). \tag{89}$$

The constant Z_1 and the mass-renormalization counterterm δm are connected with the normalization of the electron propagator. In general case the electron propagator in the momentum space is defined by two scalar functions

$$S(p) = \frac{1}{b(p^2) - a(p^2)\hat{p}} = \frac{b(p^2) + a(p^2)\hat{p}}{b^2(p^2) - a^2(p^2)p^2}.$$

If the propagator $S(p)$ possesses a pole at the point $p^2 = m_r^2$, then the normalization conditions in this point are

$$b(m_r^2) = m_r a(m_r^2), \tag{90}$$

$$a(m_r^2) + 2m_r^2 a'(m_r^2) - 2m_r b'(m_r^2) = 1. \tag{91}$$

A chiral limit for the renormalized theory means that the chiral-non-symmetric terms disappear from the renormalized Lagrangian (53):

$$Z_1 m_r + \delta m \to 0. \tag{92}$$

For transverse gauge $d_l = 0$ renormalized characteristic equation (85) has a chiral-symmetric solution at the chiral limit

$$S^{(0)} = -\frac{1}{Z_1^{(0)}\hat{p}}. \tag{93}$$

For this solution normalization conditions (90) and (91) give us the relations

$$Z_1^{(0)} = 1, \quad m_r = \delta m^{(0)} = 0.$$

Hence, this solution coincide with unrenormalized solution (73).

Renormalized equations for the first-step coefficient functions are

$$F_\lambda^{(1)}(z;x,y) = -e_r Z_1^{(0)} \int dx_1 D^c_{\lambda\mu}(z-x_1) S^{(0)}(x-x_1)\gamma_\mu S^{(0)}(x_1-y) +$$
$$+ i(e_r Z_1^{(0)})^2 \int dx_1 dy_1 D^c_{\mu\nu}(x_1-y_1) S^{(0)}(x-x_1)\gamma_\mu F_\lambda^{(1)}(z;x_1,y_1)\gamma_\nu S^{(0)}(y_1-y), \tag{94}$$

$$S_2^{(1)}(x,y;x',y') = -S^{(0)}(x-y')S^{(0)}(x'-y) +$$
$$+ i(e_r Z_1^{(0)})^2 \int dx_1 dy_1 D^c_{\mu\nu}(x_1-y_1) S^{(0)}(x-x_1)\gamma_\mu S_2^{(1)}(x_1,y_1;x',y')\gamma_\nu S^{(0)}(y_1-y), \tag{95}$$

$$S^{(1)}(x-y) = i(e_r Z_1^{(0)})^2 \int dx_1 dy_1 D^c_{\mu\nu}(x_1-y_1) S^{(0)}(x-x_1)\gamma_\mu S_2^{(1)}(x_1,y_1;y_1,y)\gamma_\nu +$$
$$+ \int dx_1 S^{(0)}(x-x_1)\Big\{[Z_1^{(1)}(i\hat{\partial} - m_r) + \delta m^{(1)}]S^{(0)}(x_1-y) +$$
$$+ie_r^2 Z_1^{(0)} \int dy_1 [2Z_1^{(1)} D^c_{\mu\nu}(x_1-y_1) + Z_1^{(0)} D^c_{\mu\nu}(x_1-y_1 \mid Z_3^{(1)})]\gamma_\mu S^{(0)}(x_1-y_1)\gamma_\nu S^{(0)}(y_1-y)\Big\} +$$
$$+i(e_r Z_1^{(0)})^2 \int dx_1 dy_1 D^c_{\mu\nu}(x_1-y_1) S^{(0)}(x-x_1)\gamma_\mu S^{(1)}(x_1-y_1)\gamma_\nu S^{(0)}(y_1-y). \quad (96)$$

As we see, the gauge invariance applies rather strict conditions: constant $Z_1^{(0)}$ is the unique renormalization constant for two first-step equations (94) and (95). In the equation for $S^{(1)}$ new renormalization constants appear: there are $Z_1^{(1)}$, $Z_3^{(1)}$ (the last one is fixed by condition (89)) and the first-step mass-renormalization counterterm $\delta m^{(1)}$.

5 Vertex

Let us go from the three-point function F_λ to the amputated three-pointer — vertex function (19). In correspondence with eq.(94) the first-step equation for the vertex is

$$\Gamma_\lambda(z;x,y) = -e\gamma_\lambda \delta(x-y)\delta(x-z) +$$
$$+ie^2 D^c_{\mu\nu}(x-y) \int dx_1 dy_1 \gamma_\mu S(x-x_1)\Gamma_\lambda(z;x_1,y_1)S(y_1-y)\gamma_\nu. \quad (97)$$

In this Section we shall use simplified notations

$$\Gamma_\lambda \equiv \Gamma_\lambda^{(1)}, \quad e \equiv e_r Z_1^{(0)}, \quad S \equiv S^{(0)}.$$

In the momentum space equation (97) has the form

$$\Gamma_\lambda(k;p) = -e\gamma_\lambda + ie^2 \int \frac{dp'}{(2\pi)^n} D^c_{\mu\nu}(p-p')\gamma_\mu S(p')\Gamma_\lambda(k;p')S(p'+k)\gamma_\nu. \quad (98)$$

Here k is the photon momentum, p is the electron momentum (see definition (87) of the Fourier-image of three-pointer). Equation (98) is known in literature as Edwards equation [15]. Note again that in our approach this equation is not a result of arbitrary truncation, but is a consequence of the iteration scheme equations. Electron propagator S in this equation is a solution of characteristic equation (85) which defines the leading vacuum approximation.

We consider a solution of vertex equation (97) at small k with above mentioned chiral-symmetric propagator (93) in the transverse gauge $d_l = 0$. To solve this equation it is convenient to introduce a function $\Phi_\lambda(k;p)$ which is defined by the relation

$$\Gamma_\lambda(k;p) \equiv \hat{p}\Phi_\lambda(k;p)(\hat{p}+\hat{k}).$$

Expand Φ_λ near $k=0$

$$\Phi_\lambda(k;p) = \Phi_\lambda(p) + k_\rho \Phi_{\lambda\rho}(p) + \cdots$$

Here

$$\Phi_\lambda(p) \equiv \Phi_\lambda(0;p), \quad \Phi_{\lambda\rho}(p) \equiv \partial_\rho^k \Phi_\lambda(k;p)|_{k=0}.$$

At that

$$\Gamma_{\lambda\rho}(p) \equiv \partial_\rho^k \Gamma_\lambda(k;p)|_{k=0} = \hat{p}\Phi_{\lambda\rho}(p)\hat{p} + \hat{p}\Phi_\lambda(p)\gamma_\rho. \quad (99)$$

Equations for $\Phi_\lambda(p)$ and $\Phi_{\lambda\rho}$ are

$$\hat{p}\Phi_\lambda(p)\hat{p} = -e\gamma_\lambda + ie_r^2 \int \frac{dp'}{(2\pi)^n} D^c_{\mu\nu}(p-p')\gamma_\mu \Phi_\lambda(p')\gamma_\nu \tag{100}$$

and

$$\hat{p}\Phi_{\lambda\rho}(p)\hat{p} = -\hat{p}\Phi_\lambda(p)\gamma_\rho + ie_r^2 \int \frac{dp'}{(2\pi)^n} D^c_{\mu\nu}(p-p')\gamma_\mu \Phi_{\lambda\rho}(p')\gamma_\nu. \tag{101}$$

It is more convenient to solve eq.(100) in the coordinate space:

$$-\hat{\partial}\Phi_\lambda(x)\hat{\partial} = -e\gamma_\lambda\delta(x) + ie_r^2 D^c_{\mu\nu}(x)\gamma_\mu\Phi_\lambda(x)\gamma_\nu. \tag{102}$$

Here $D^c_{\mu\nu}(x)$ is defined by eq.(74). Due to the property of "$\hat{x}$-transversality" (see (75)) all iterations of eq.(102) turn into zero. Indeed, a zero approximation is

$$\Phi^0_\lambda(p) = -e\frac{1}{\hat{p}}\gamma_\lambda\frac{1}{\hat{p}} = e\partial^p_\lambda\frac{1}{\hat{p}}$$

and in x-space

$$\Phi^0_\lambda(x) = e\frac{ie^{-i\pi n/2}\Gamma(n/2)}{2\pi^{n/2}}\frac{x_\lambda\hat{x}}{(x^2-i0)^{n/2}} \sim \hat{x}.$$

Consequently

$$\Phi_\lambda = \Phi^0_\lambda$$

and

$$\Gamma_\lambda(p) = -e\gamma_\lambda. \tag{103}$$

Let us go to solving eq.(101) for $\Phi_{\lambda\rho}$. (We consider now the four-dimensional case $n = 4$.) This equation is also more convenient for solving in the coordinate space, where it has the form

$$-\hat{\partial}\Phi_{\lambda\rho}(x)\hat{\partial} = \frac{e}{2\pi^2}\frac{\gamma_\lambda\hat{x}\gamma_\rho}{(x^2-i0)^2} + ie_r^2 D^c_{\mu\nu}(x)\gamma_\mu\Phi_{\lambda\rho}(x)\gamma_\nu. \tag{104}$$

(We have taken into account the result of solving equation for $\Phi_\lambda(p)$.) Here

$$D^c_{\mu\nu}(x) = \frac{1}{4i\pi^2(x^2-i0)}(\frac{g_{\mu\nu}}{2} + \frac{x_\mu x_\nu}{x^2-i0}). \tag{105}$$

Expanding $\Phi_{\lambda\rho}$ over spinor structures

$$\Phi_{\lambda\rho} = \Phi_{\lambda\rho\sigma}\gamma_\sigma + \Phi^5_{\lambda\rho\sigma}\gamma_5\gamma_\sigma$$

we obtain two equations

$$(g_{\sigma\tau}\partial^2 - 2\partial_\sigma\partial_\tau)\Phi_{\lambda\rho\sigma}(x) = \frac{e}{2\pi^2}\frac{g_{\lambda\tau}x_\rho - g_{\lambda\rho}x_\tau + g_{\rho\tau}x_\lambda}{(x^2-i0)^2} - \\ -\frac{2\alpha_r}{\pi}\frac{1}{x^2-i0}(g_{\sigma\tau} - \frac{x_\sigma x_\tau}{x^2-i0})\Phi_{\lambda\rho\sigma}(x), \tag{106}$$

$$(g_{\sigma\tau}\partial^2 - 2\partial_\sigma\partial_\tau)\Phi^5_{\lambda\rho\sigma}(x) = \frac{ie}{2\pi^2}\epsilon_{\lambda\rho\sigma\tau}\frac{x_\sigma}{(x^2-i0)^2} - \frac{2\alpha_r}{\pi}\frac{1}{x^2-i0}(g_{\sigma\tau} - \frac{x_\sigma x_\tau}{x^2-i0})\Phi^5_{\lambda\rho\sigma}(x). \tag{107}$$

Here

$$\alpha_r = \frac{e_r^2}{4\pi}.$$

As for equation for Φ_λ iterations of eq. (106) for $\Phi_{\lambda\rho\sigma}$ turn into zero, and its solution is

$$\Phi_{\lambda\rho\sigma}(x) = -\frac{e}{4\pi^2}\frac{x_\lambda x_\rho x_\sigma}{(x^2 - i0)^2}.$$

Equation (107) for $\Phi^5_{\lambda\rho\sigma}$ possesses a solution

$$\Phi^5_{\lambda\rho\sigma}(x) = \frac{ie}{4\pi(2\pi - \alpha_r)}\epsilon_{\lambda\rho\sigma\tau}\frac{x_\tau}{x^2 - i0}.$$

Going to the momentum space we finally obtain for $\Gamma_{\lambda\rho}$ (see (99)):

$$\Gamma_{\lambda\rho}(p) = ie\frac{\alpha_r}{2\pi - \alpha_r}\epsilon_{\lambda\rho\sigma\tau}\gamma_5\gamma_\sigma\frac{p_\tau}{p^2 + i0}. \tag{108}$$

Matrix element of vertex function $\Gamma_\lambda(k;p)$ at small k_μ defines two formfactors f_1 and f_2

$$\bar{u}(q)\Gamma_\lambda u(p) \simeq -e_r f_1 \bar{u}(q)\gamma_\lambda u(p) - \frac{e_r}{2m_r}f_2\bar{u}(q)[\gamma_\lambda, \hat{k}]u(p), \tag{109}$$

which connected correspondingly with a physical charge and a magnetic moment (see, for example, [19]). Here $u(p)$ is a solution of Dirac equation $(\hat{p} - m_r)u(p) = 0$. Comparing with calculated vertex we obtain at $k = 0$

$$e_r f_1 = e = e_r Z_1^{(0)}.$$

For the normalization on the charge e_r we put $f_1 = 1$ and, consequently,

$$Z_1^{(0)} = 1.$$

This very important fact means that the normalization of the vertex function on the renormalized charge gives in our approach the same value of renormalization constant Z_1 as wave-function renormalization condition (91), i.e. our calculational scheme is consistent with requirements which imposed by gauge invariance.

As for the second formfactor f_2, which defines a correction to the magnetic moment, note, that the massless particle, of course, has no magnetic moment. However, the calculated first term of vertex function expansion in k_μ (see (108)) permits making a nonperturbative estimate for anomalous magnetic moment in chiral limit.

Let us define the problem more exactly. In expansion of f_2 in orders of α_r

$$f_2 = C_1\frac{\alpha_r}{\pi} + C_2\Big(\frac{\alpha_r}{\pi}\Big)^2 + \cdots$$

coefficients C_i, starting with the second one, are mass-depended. (So, they have even different signs for muon and electron, see, for example, [20].) Nevertheless, one can, in principle, to consider a peculiar "chiral limit" for these coefficient, i.e. their values at $m_r \to 0$. It is this estimate which we are talking about. To obtain this estimate let us make an identical transformation of the second term in eq.(109)

$$\frac{1}{m_r}\bar{u}(q)[\gamma_\lambda, \hat{k}]u(p) = \frac{1}{p^2}\bar{u}(q)[\gamma_\lambda, \hat{k}]\hat{p}u(p). \tag{110}$$

For massive particles it is an identity due to Dirac equation for $u(p)$. However, for r.h.s. of eq.(110) one can use result (108) which was obtained for massless particle. Comparing coefficients at $\epsilon_{\lambda\rho\sigma\tau}$, we obtain

$$f_2 = \frac{\alpha_r}{2\pi - \alpha_r}. \tag{111}$$

The first term of the expansion in α_r coincides with the Schwinger correction (see, for example, [20]). At formal limit $\alpha_r \to \infty$ the full magnetic moment in correspondence with eq.(111) turn to zero. It is interesting to note, that similar screening of chromomagnetic moments of quarks was pointed out in some models of nonperturbative quantum chromodynamics [21] and also in relativistic quarkonium model [22].

6 Dynamical chiral symmetry breaking

In this Section we consider a problem of dynamical chiral symmetry breaking (DCSB) for QED in the framework of proposed calculational scheme over the nonperturbative vacuum. Note, that our present consideration does not cover all aspects of this complicated problem, in particular, we do not consider a question of connection of DCSB with triviality problem. A consideration of this and a number of other questions imposes an investigation of the first-step equations for higher Green functions. We limit ourselves to investigation of the leading-approximation equation.

A renormalized equation of leading approximation for the electron propagator has the form

$$S^{-1}(x) = (m - Zi\hat{\partial})\delta(x) - ie^2 D^c_{\mu\nu}(x)\gamma_\mu S(x)\gamma_\nu. \tag{112}$$

Here and everywhere in this Section we denote

$$S \equiv S^{(0)}, \;\; Z \equiv Z_1^{(0)}, \;\; m \equiv Z_1^{(0)} m_r + \delta m^{(0)}, \;\; e \equiv e_r Z_1^{(0)},$$

and the free propagator $D^c_{\mu\nu}(x)$ is defined by eq.(74).

In general case (if one does not consider parity breaking solutions) S can be represented as

$$S = i\hat{\partial}A + B,$$

where A and B are scalar functions of unique variable. Correspondingly, for the inverse propagator we have

$$S^{-1} = -i\hat{\partial}a + b,$$

where in the momentum space the functions a and b are connected with the functions A and B by relations

$$A = \frac{a}{b^2 - a^2p^2}, \;\; B = \frac{b}{b^2 - a^2p^2}. \tag{113}$$

Taking into account eq.(74) and properties of Dirac matrices, characteristic equation (112) in the coordinate space can be represented as a system of two equations

$$i\hat{\partial}a = Zi\hat{\partial}\delta(x) - 2\alpha d_l \frac{e^{-i\pi n/2}\Gamma(\frac{n}{2})}{[\pi(x^2 - i0)]^{n/2-1}} \cdot i\hat{\partial}A, \tag{114}$$

$$b = m\delta(x) + \alpha(1 - n - d_l)\frac{e^{-i\pi n/2}\Gamma(\frac{n}{2} - 1)}{[\pi(x^2 - i0)]^{n/2-1}} \cdot B. \tag{115}$$

Here

$$\alpha \equiv \frac{e^2}{4\pi}.$$

As is evident from eqs. (114)-(115), two special gauges exist for characteristic equation (112)

1)the transverse Landau gauge $d_l = 0$. In this gauge eq.(114) becomes trivial and possesses a solution $a = Z\delta(x)$.

2)the gauge $d_l = 1-n$. In this gauge[3] $b = m\delta(x)$ *in case if the product* $B(x)\cdot(x^2-i0)^{1-n/2}$ *is well-defined in a sense of generalized functions, i.e., divergences are absent.* Otherwise, in contrast with the Landau gauge, we have an uncertainty of the type $0\cdot\infty$. Its evaluation is performed by a regularization, and we can only say that in this gauge $b(p^2) = \text{const}$.

Below in this Section we shall consider the four-dimensional case. Multiplying eqs.(114) and (115) by x^2, we obtain the equations

$$x^2 i\hat{\partial}a = -\frac{2\alpha}{\pi} d_l i\hat{\partial}A, \tag{116}$$

$$x^2 b = -\frac{\alpha}{\pi}(3 + d_l)B. \tag{117}$$

Such a multiplication can be considered as some regularization of potentially singular products in the r.h.s. of eqs. (114) and (115). More, going to p-space and taking into account eq.(113), we obtain for $a(p^2)$ and $b(p^2)$ a system of ordinary differential equations

$$t\frac{d^2a}{dt^2} + 3\frac{da}{dt} = \frac{\alpha}{2\pi} d_l \frac{a}{b^2 - a^2 t}, \tag{118}$$

$$t\frac{d^2b}{dt^2} + 2\frac{db}{dt} = \frac{\alpha}{4\pi}(3 + d_l)\frac{b}{b^2 - a^2 t}. \tag{119}$$

Here $t = p^2$.

This very complicated system of nonlinear differential equations has the simplest form in the transverse Landau gauge $d_l = 0$. Below we shall exploit this gauge. As was mentioned earlier, in this gauge $a = Z$, and system (118)-(119) is reduced to the equation for b

$$t\frac{d^2b}{dt^2} + 2\frac{db}{dt} = \frac{3\alpha}{4\pi}\frac{b}{b^2 - Z^2 t}. \tag{120}$$

This equation always has the trivial solution $b \equiv 0$, which corresponds to above mentioned chiral-symmetric solution (93). The existence of non-trivial solutions is also possible, and their asymptotics is not difficult to find. Really, at $t \to \infty$ two variants are possible:

a) $b^2 \sim t$;

b) $b^2 \ll \mid t \mid$.

It is not difficult to prove that the third possibility $b^2 \gg \mid t \mid$ up to logarithms leads to the variant a).

Consider the variant b). Then in the ultraviolet region $t \to \infty$ equation (120) is reduced to Euler equation, and an asymptotic behavior of $b(t)$ is

$$b \sim t^{-1/2(1-\sqrt{1-3\alpha_r/\pi})} \ \text{ at } \ \alpha_r < \pi/3, \tag{121}$$

[3]At $n = 4$ it is Solovev-Yennie gauge [23].

$$b \sim t^{-1/2} \log t \ \text{ at } \ \alpha_r = \pi/3 \tag{122}$$

and

$$b \sim t^{-1/2} \sin \{ \frac{\sqrt{3\alpha_r/\pi - 1}}{2} \log t \} \ \text{ at } \ \alpha_r > \pi/3. \tag{123}$$

Note, that here $\alpha_r \equiv \frac{e_r^2}{4\pi}$ is the renormalized fine structure constant.

The existence of a critical value

$$\alpha_r = \alpha_c \equiv \pi/3 \tag{124}$$

should be pointed out, at which a change of ultraviolet behavior regime takes place. The existence of this critical point was firstly pointed in work [24]. In works [4] such regime transition was connected with DCSB in QED. (See also [17],[7] where this approach is discussed in detail and the extensive bibliography is given. For latest developments of this approach see [25].) In all cited works a scheme with ultraviolet cutoff in Euclidean space was exploited. A main calculational ansatz was a linearization of eq. (120), which consists in an approximation [4]

$$\frac{b}{b^2 - t} \approx \frac{b}{m^2 - t},$$

in the r.h.s. of eq.(120). Here $m \equiv b(0)$. After such a linearization the equation for $b(t)$ becomes a hypergeometrical equation. Boundary conditions for this equation are defined from the integral equation which is eq.(72) in the momentum space [5]. An asymptotic behavior of solutions of linearized equation is given by the same formulae (121)-(123), and this fact is the main argument in favor of the linearized version (at least in the ultraviolet region). These solutions, of course, essentially depend on a cutoff parameter which is included in the boundary conditions. Nevertheless, one was succeeded in demonstrating the existence of a phase transition at the critical point $\alpha_c = \pi/3$. This transition corresponds to DCSB. In the chiral limit under the critical point (weak coupling) only the chiral-symmetric solution exists, but beyond the critical point (strong coupling), when the ultraviolet asymptotics becomes oscillating, a solution with dynamical mass $b \neq 0$ arises, i.e., the spontaneous breaking of chiral symmetry takes a place.

We consider this problem in the pseudoeuclidean Minkowski space for renormalized equation (112).[6] At that we shall use another approach to the investigation of the propagator equation, which, nevertheless, is similar to the approach of above mentioned works and also based on a linearization of nonlinear equation (112). Since we work with renormalized theory, our results are cutoff-independent. In general they are consistent with the results of linearized unrenormalized theory in Euclidean space up to the unique exception: in pre-critical region $\alpha_r < \alpha_c$ DCSB is also possible, but under some condition on value of α_r. An investigation of this condition requires studying of equation for the three-point function with the nonperturbative electron propagator and goes out of the framework of this work.

We shall solve equation (112) in transverse gauge $d_l = 0$ at $m \neq 0$ by iterations and a foundation for iterative solution will be an exact solution at $m = 0$

[4] Note, that in the cutoff scheme $Z = 1$.

[5] An essential point at the formulation of these boundary conditions is exploiting of Euclidean version of the theory.

[6] This point seems to be essential since detailed investigations of unrenormalized equation (72) in Euclidean space at $m = 0$ (see, for example, [26]) demonstrate the existence of non-pole complex singularity for its solutions, and, hence, the Euclidean rotation seems to be a problem.

$$S_0^{-1} = -Z\hat{p}.$$

At that the linearization consists in rather natural procedure from the point of view of the iterative solution: starting from the representation of the inverse propagator

$$S^{-1} = S_0^{-1} + \Sigma$$

we approximate

$$S = [S_0^{-1} + \Sigma]^{-1} \approx S_0 - S_0 \star \Sigma \star S_0 = -\frac{\hat{p}}{Z(p^2 + i0)} - \frac{\hat{p}\Sigma\hat{p}}{Z^2(p^2 + i0)^2}.$$

It is clear that such an approximation can be fully valid only in the ultraviolet region $\mid p^2 \mid \gg \mu^2$, where μ^2 is some scale which can be treated as an infrared cutoff. Since a nonperturbative region of QED is the ultraviolet region, the natural supposition is in the following: the ultraviolet behavior is crucial for basic nonperturbative effects including DCSB.

Using the property of $\hat{x}$-transversality (see (75)) it is easy to prove $\Sigma = 1 \cdot \sigma$, and for scalar function σ in the coordinate space we obtain the equation

$$\sigma(x^2) = m\delta(x) + \frac{3\alpha_r}{\pi}\frac{1}{x^2 - i0} \cdot \phi(x^2), \tag{125}$$

where

$$\phi(p^2) \equiv \frac{\sigma(p^2)}{p^2 + i0}.$$

The four-dimensional delta-function $\delta(x)$ in the Minkowski space is a Lorentz-invariant distribution (see, for example, [27]), and to solve eq.(125) it is convenient to use a representation of the delta function as a limit of a sequence of functions in x^2-variable. As such a representation we choose the formula

$$\delta^4(x) = \frac{i}{\pi^2}\lim_{\lambda \to 0} \lambda(x^2 - i0)^{\lambda - 2} \tag{126}$$

which can be easy proved with the Fourier-transform of function $(x^2 - i0)^{\lambda-2}$ (see, for example, [28]).

A partial solution (a solution of inhomogeneous equation) will be searched as

$$\sigma_0(x^2) = C_0(x^2 - i0)^{\lambda - 2}$$

at small λ. As we shall see below, $1/\lambda$ plays the role of a regularization parameter. Making the corresponding Fourier-transform (see [28]), we obtain

$$\phi_0(x^2) = \frac{C_0}{4\lambda(1 - \lambda)}(x^2 - i0)^{\lambda - 1}, \tag{127}$$

and, substituting into eq. (125), we finally obtain

$$C_0 = -\frac{4im}{3\pi\alpha_r}\lambda^2 + \mathcal{O}(\lambda^3).$$

Consequently, the solution of the inhomogeneous equation is

$$\sigma_0(p^2) = -\frac{4\pi m\lambda}{3\alpha_r}. \tag{128}$$

The full solution of eq.(125) is a sum of σ_0 and a general solution of homogeneous equation. A solution of homogeneous equation is sought for in the form

$$\bar{\sigma}(x^2) = C(x^2 - i0)^{\beta-2}.$$

Hence $\bar{\phi}(x^2)$ is defined by the same formula (127) with the substitution $\lambda \to \beta$. For parameter β we obtain the equation

$$\beta(1-\beta) = \frac{3\alpha_r}{4\pi}. \tag{129}$$

If one chooses a solution with $C = 0$, i.e., limits oneself to consideration of partial solution (128), then normalization conditions (90) and (91) give [7]

$$\delta m^{(0)} = \frac{3\alpha_r}{4\pi\lambda} m_r, \quad Z = 1$$

and, correspondingly, in this case

$$S^{-1} = -\hat{p} + m_r.$$

In the chiral limit (see (92)), when $m \to 0$, we obtain $m_r = 0$, therefore this solution is a chiral-symmetric one.

But if one permits solutions with $C \neq 0$, the situation is essentially changed. Note at once, that the solutions of the homogeneous equation in p-space are singular at $p = 0$, but, due to the essence of the approximation made, all subsequent formulae can be interpreted, as was pointed out above, only at $\mid p^2 \mid \gg \mu^2$, where μ^2 is an infrared cutoff.

So, in dependence of value of parameter α_r, we have for electron mass function:

1) At $\alpha_r < \alpha_c$ (weak coupling)

$$b(p^2) = -\frac{4\pi m\lambda}{3\alpha_r} + C(p^2 + i0)^{-\beta}, \tag{130}$$

where $\beta = \frac{1}{2}(1 - \sqrt{1 - \alpha_r/\alpha_c})$. [8]

2) At $\alpha_r = \alpha_c = \pi/3$

$$b(p^2) = -\frac{4\pi m\lambda}{3\alpha_r} + \frac{C \log \frac{p^2+i0}{M^2}}{(p^2+i0)^{1/2}}. \tag{131}$$

3) At $\alpha_r > \alpha_c$ (strong coupling)

$$b(p^2) = -\frac{4\pi m\lambda}{3\alpha_r} + \frac{C \sin\{\frac{\omega}{2} \log \frac{p^2+i0}{M^2}\}}{(p^2+i0)^{1/2}}. \tag{132}$$

[7] The formula for counterterm δm demonstrates a role of λ as a parameter of regularization which removes ultraviolet divergence. Therefore, the representation of delta-function by formula (126) can be considered as a special analytical regularization

[8] We take a root of eq.(129) which corresponds to the ultraviolet asymptotics of exact solution, see (121).

Here

$$\omega = \sqrt{\frac{\alpha_r}{\alpha_c} - 1}.$$

$a = Z$ for any of three cases.

Below we consider the chiral limit $m \equiv Zm_r + \delta m^{(0)} = 0$. Solution parameters are fixed by normalization conditions (90)-(91) of the electron propagator. If (as in ultraviolet-cutoff scheme [4],[17], [25]) one takes $Z = 1$, then the normalization conditions become

$$b(m_r^2) = m_r, \quad m_r b'(m_r^2) = 0. \tag{133}$$

In the weak coupling case ($\alpha_r < \alpha_c$) it follows from the normalization conditions that either $C = 0$ or $m_r = 0$, i.e. in this case DCSB-solutions are absent. In the critical case $\alpha_r = \alpha_c$ a solution with $m_r \neq 0$ is possible which has the form

$$b(p^2) = \frac{m_r^2}{(p^2 + i0)^{1/2}}\Big(1 + \frac{1}{2}\log\frac{p^2 + i0}{m_r^2}\Big). \tag{134}$$

At $\alpha_r > \alpha_c$ (strong coupling) a DCSB-solution also exists:

$$b(p^2) = \frac{m_r^2}{(p^2 + i0)^{1/2}}\Big[\cos\Big\{\frac{\omega}{2}\log\frac{p^2 + i0}{m_r^2}\Big\} + \frac{1}{\omega}\sin\Big\{\frac{\omega}{2}\log\frac{p^2 + i0}{m_r^2}\Big\}\Big]. \tag{135}$$

As we can see, these results correspond to those of the unrenormalized theory with a cutoff in the Euclidean space (see [4],[17],[25]). However for the renormalized theory we have no any prior reason to set $Z = 1$. If one refuses this condition, the results in the weak coupling region can be essentially different. Really, at $Z \neq 1$ the normalization conditions are

$$b(m_r^2) = m_r Z, \quad 2m_r b'(m_r^2) = Z - 1. \tag{136}$$

At $\alpha_r < \alpha_c$, in contrast with the case $Z = 1$, the existence of DCSB-solution becomes possible. It has the form

$$a = Z = \frac{1}{1 + 2\beta}, \quad b = Zm_r\Big(\frac{m_r^2}{p^2 + i0}\Big)^{\beta}. \tag{137}$$

At $\alpha_r = \alpha_c$ a solution is

$$b(p^2) = \frac{m_r^2}{(p^2 + i0)^{1/2}}\Big(Z + (Z - \frac{1}{2})\log\frac{p^2 + i0}{m_r^2}\Big). \tag{138}$$

and, at last, at $\alpha_r > \alpha_c$

$$b(p^2) = \frac{m_r^2}{(p^2 + i0)^{1/2}}\Big[Z\cos\Big\{\frac{\omega}{2}\log\frac{p^2 + i0}{m_r^2}\Big\} + \frac{1}{\omega}(2Z - 1)\sin\Big\{\frac{\omega}{2}\log\frac{p^2 + i0}{m_r^2}\Big\}\Big]. \tag{139}$$

($a = Z$ for two last cases, Z is arbitrary.)

Therefore, with taking into account the renormalization freedom DCSB becomes possible in the weak coupling region which corresponds to physical sector of the theory. At that, however, an important limitation connected with the gauge invariance arises. The necessity for such a limitation follows also from a consideration of switch-off-interaction limit. Indeed, it follows from eq.(137) that at $\alpha_r \to 0 \quad b \to m_r + \mathcal{O}(\alpha_r)$, i.e. either $m_r = 0$, or DCSB does

not disappear in the switch-off-interaction limit. The last possibility looks quite strange. A way out of this situation can be in the following: all the picture in the weak coupling region is realized only for some finite values of α_r. A condition for such values is a normalization of the vertex on physical charge e_r:

$$\Gamma_\mu(k=0; p^2 = m_r^2 \mid \alpha_r, Z) = -e_r\gamma_\mu. \tag{140}$$

Due to the gauge invariance the renormalization constant Z is the same as for the electron propagator equation, and substituting the value $Z = Z(\alpha_r)$ from eq.(137) we shall obtain an equation for α_r. Solutions of this equation will define "permitted" values of the charge. A realization of this program requires solving an equation for the vertex function with propagator (137).

At $\alpha_r \geq \alpha$ the situation changes in principle: in this case Z (at the leading approximation) is arbitrary, and the normalization on the physical charge fixes Z, but does not lead to additional limitations on values of α_r.

Conclusion

Above results give us a reasoning to consider the calculations over the nonperturbative vacuum as an quite adequate scheme for calculations of nonperturbative effects in QED. A formulation of system of equations for Green functions at any step of this calculational scheme is technically as simple as the coupling constant perturbation theory [9] and does not require anything except of the "stupid differentiation". The equations themselves, of course, are much more complicated, but, as we can see from the calculation of the vertex function for chiral-symmetric solution (Section 5), they possess much more simple solutions in comparison with analogous equations for Green functions in nonperturbative approximations which based on a more or less arbitrary truncation of Dyson equations.

As regards the calculations of Section 6, which concern the DCSB problem, the one of main questions concerning the linearized equation for electron propagator is a problem of a gauge dependence of the results, in particular, of a gauge dependence of the critical constant α_c (see [17] and references therein). In our treatment this equation is not a result of an arbitrary truncation of Dyson equations but a consistent part of the iteration scheme, and this question becomes unavoidable. In this connection we note that the linearization seems to be substantiate in Landau gauge only, and for other gauges the nonlinear effects scarcely can be taken into account with such simple ansatz. As has been mentioned above, the results of Section 6 in strong coupling region coincide in essence to the results of so-called rainbow approximation, or "quenched QED with bare vertex" (see [17], [25]). It is no wonder since the basic equation for electron propagator is the same, and a difference consists in the regularization and renormalization scheme. Generally, quenched QED is an approximation of QED with an effect of pair creation being neglected. For the electron propagator this approximation permits more general form of the vertex function. For example, the use of Curtis-Pennington vertex [29] solves the problem of gauge dependence of critical coupling α_c, though is not solve all the problems in strong coupling region (see [30] and references

[9]Note, the coupling constant perturbation theory can be considered as a partial case of the general iteration scheme based on principles of Section 4. To obtain a perturbation theory series it is enough consider the same iteration scheme with single Grassman fermion sources over perturbative vacuum which corresponds to leading approximation $G^{(0)} = 1$ (see also [9]-[10]).

therein). Our approach is not an approximation of QED but an iteration scheme, so we have not a possibility for such generalization of leading order equation for electron propagator. As a general remark, note, that the strong-coupled four-dimensional QED is of pure theoretical interest as a model gauge theory for investigation of chiral symmetry breaking phenomenon. The results of various approaches (such as worldline variational approach, high orders of perturbation theory, quenched approximation etc.) essentially differ in strong coupling region (see [31] for review), which indicates our present poor understanding of this problem.

In closing we shortly discuss a problem of DCSB for quantum chromodynamics (QCD). In contrast to QED, where nonperturbative effects (and DCSB among them) are defined by ultraviolet region, for asymptotically-free QCD a nonperturbative region is the low-energy infrared region. At that a mechanism of DCSB in QCD can be even more simple in comparison with QED, since effective taking into account of gluon self-action in the infrared region inevitably leads to appearance of new dimensional parameters.

As a simple example of a realization of similar DCSB mechanism by infrared singularities consider two-dimensional QED. The analogy among the nonperturbative effects of two-dimensional QED and those of four-dimensional QCD was repeatedly pointed out. In spite of certainly limited nature of such analogy, an investigation of the nonperturbative dynamics on example of much more technically simple QED can be useful for understanding of QCD in the nonperturbative region.

At $n = 2$ the free propagator $D^c_{\mu\nu}(x)$ (see (74)) entering equation (112) in general case is infrared-singular, and it is necessary to introduce some infrared cutoff. However, a gauge exists for which such a cutoff is unnecessary. This is above mentioned infrared-finite gauge $d_l = 1 - n = -1$. As follows from eq.(74), in this gauge

$$D^c_{\mu\nu}(x) = \frac{i}{2\pi}\frac{x_\mu x_\nu}{x^2 - i0},$$

and the equation for the electron propagator is simply

$$S^{-1} = (S^c)^{-1} + 2\alpha S. \tag{141}$$

In the momentum space eq.(141) is reduced to a system of algebraic equations, and among its solutions a DCSB-solution exists. Surely, in the two-dimensional theory the spontaneous breaking of continuous chiral symmetry is not realized since such a state is unstable in correspondence with Mermin-Wagner-Coleman theorem. Nevertheless, the fact itself is remarkable from the point of view of above mentioned analogy with four-dimensional QCD.

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